rijksuniversiteit groningen

Regularizing turbulent flow

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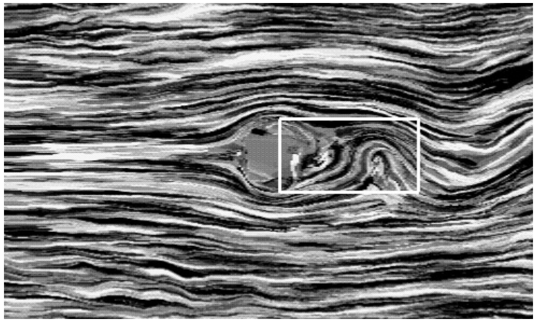
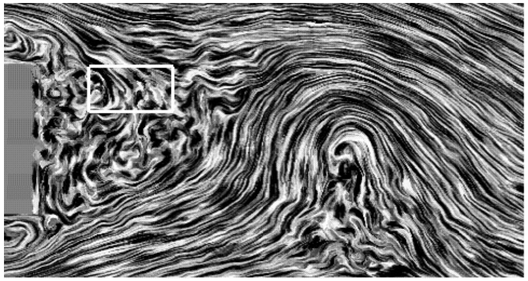
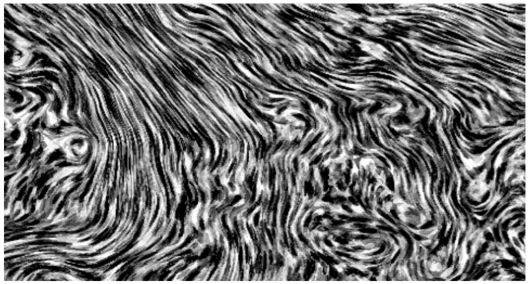
Turbulence: a multi-scale phenomena

Big whorls have little whorls
That feed on their velocity,
And little whorls have lesser whorls
And so on to viscosity Lewis F. Richardson







Energy cascade

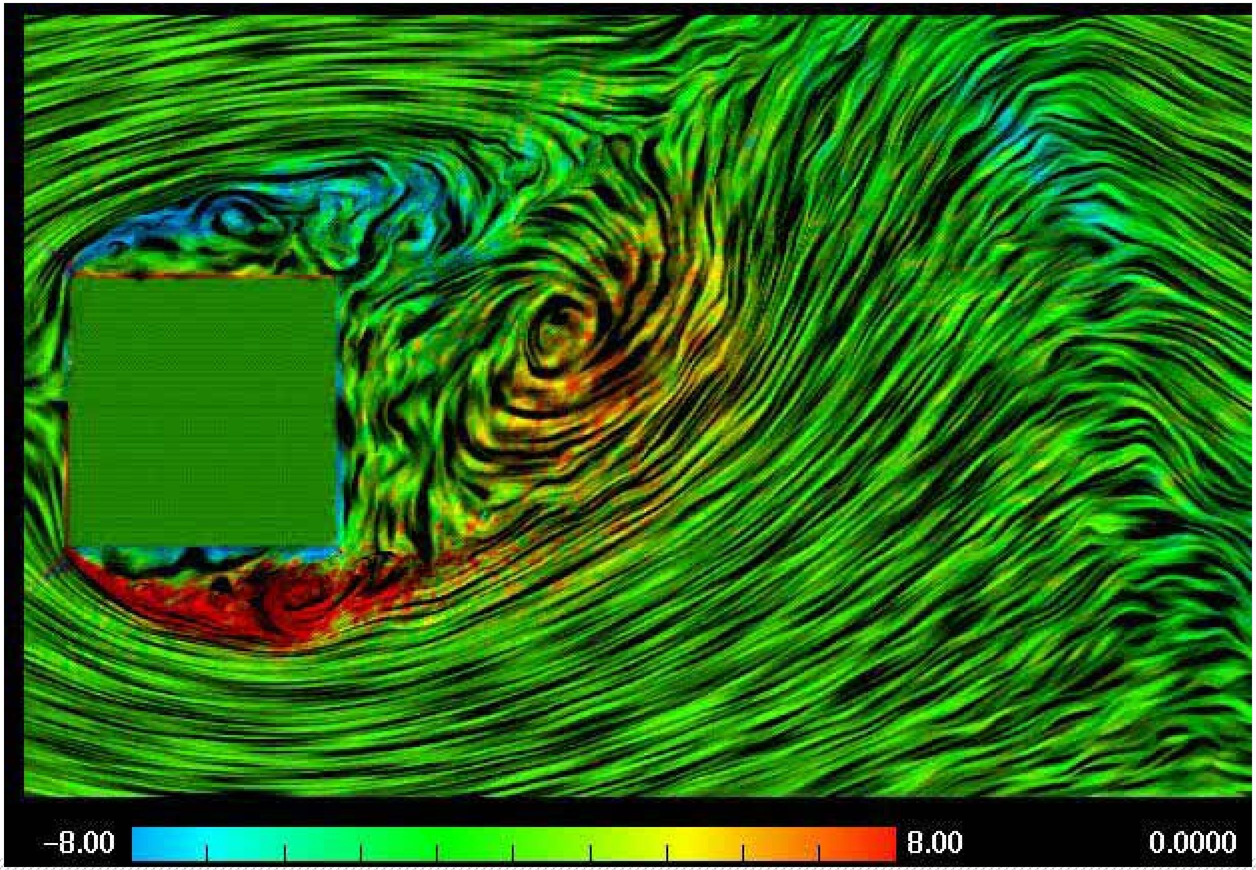
Large scales of motion are created by some driving mechanism that feeds energy to the fluid

Smaller scales are convected, drawing energy from the large ones

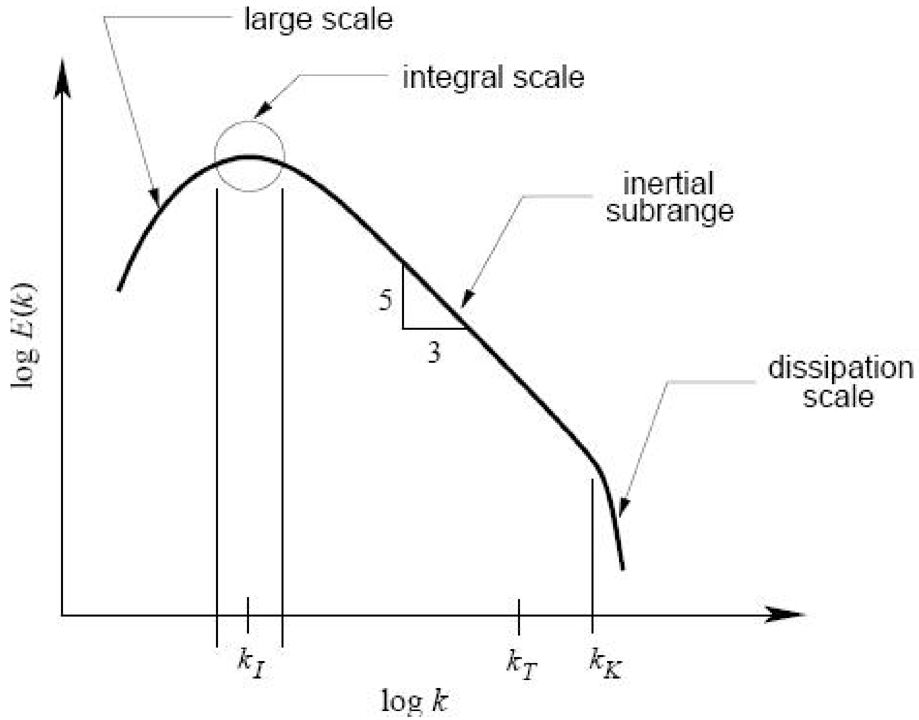
This process continues on ever smaller scales

On the smallest scales, dissipation destroys eddies and converts their kinetic energy to thermal energy

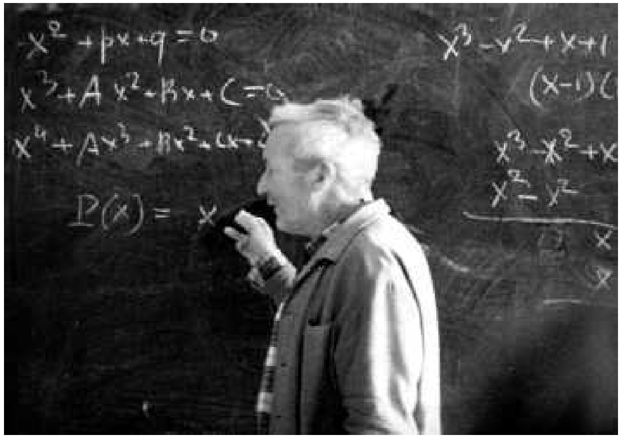
3D turbulent flow past square cylinder (Re=22,000)



Kolmogorov's K41 hypotheses



Mathematical justification: Foias et al 2001



Challenges	Re
Cyclist	100,000
Car	>1,000,000
Swimmer	3,000,000
Airplane	10,000,000
Shark	30,000,000

Work propto Re^3

Navier-Stokes equations

$$\partial_t \boldsymbol{u} + \mathcal{C}(\boldsymbol{u}, \boldsymbol{u}) + \mathcal{D}(\boldsymbol{u}) + \nabla p = 0$$

$$\mathcal{C}(\boldsymbol{u}, \boldsymbol{v}) = (\boldsymbol{u} \cdot \nabla)\boldsymbol{v}$$

Regularization

$$\partial_t \boldsymbol{u} + \tilde{\mathcal{C}}(\boldsymbol{u}, \boldsymbol{u}) + \mathcal{D}(\boldsymbol{u}) + \nabla p = 0$$

Examples

Leray: $\tilde{\mathcal{C}}(\boldsymbol{u}, \boldsymbol{v}) = \mathcal{C}(\overline{\boldsymbol{u}}, \boldsymbol{v})$

NS-α*:* $\tilde{\mathcal{C}}_r(\boldsymbol{u}, \boldsymbol{v}) = \mathcal{C}_r(\boldsymbol{u}, \overline{\boldsymbol{v}}) = (\nabla \times \boldsymbol{u}) \times \overline{\boldsymbol{v}}$

Leray regularization

Leray (1934): any filtering is sufficient to guarantee existence, uniqueness and regularity

$\Rightarrow$ energy cascade stops at a certain scale of motion

Cheskidov et al. (2005): complexity of 3D Leray model lies between 2D and 3D Navier-Stokes

Large-eddy simulation

$$\partial_t \overline{\boldsymbol{u}} + \mathcal{C}(\overline{\boldsymbol{u}}, \overline{\boldsymbol{u}}) + \mathcal{D}(\overline{\boldsymbol{u}}) + \nabla \overline{p} = \text{model}(\overline{\boldsymbol{u}})$$

$$\text{model}(\overline{\boldsymbol{u}}) \approx \mathcal{C}(\overline{\boldsymbol{u}}, \overline{\boldsymbol{u}}) - \overline{\mathcal{C}(\boldsymbol{u}, \boldsymbol{u})}$$

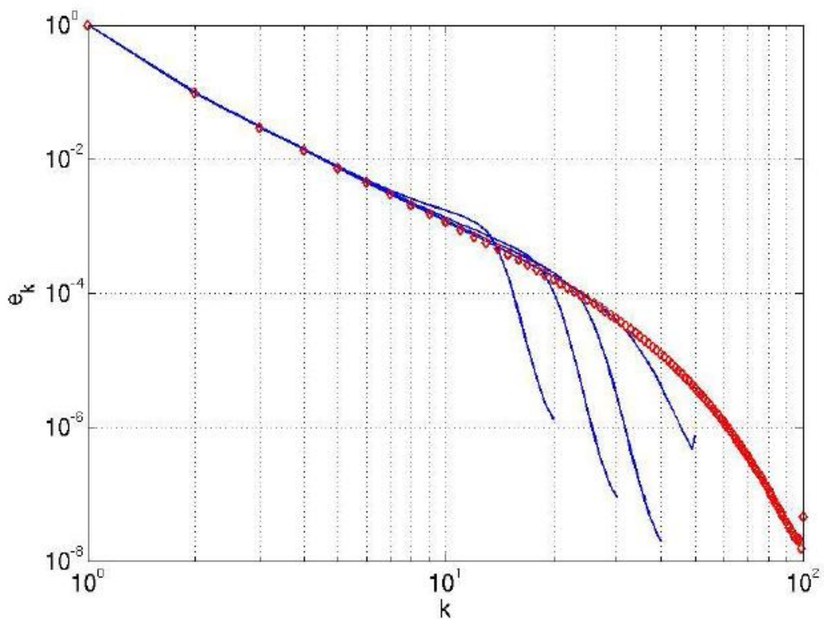
Regularization model

$$\partial_t \overline{\boldsymbol{u}} + \mathcal{C}(\overline{\boldsymbol{u}}, \overline{\boldsymbol{u}}) + \mathcal{D}(\overline{\boldsymbol{u}}) + \nabla \overline{p} =$$

$$\mathcal{C}(\overline{\boldsymbol{u}}, \overline{\boldsymbol{u}}) - \overline{\tilde{\mathcal{C}}(\boldsymbol{u}, \boldsymbol{u})} = \text{model}(\overline{\boldsymbol{u}})$$

How to alter the non-linearity?

Goal:



Preserve:

- *inertial subrange*
- *symmetry & conservation*
- *transformation properties*
- *etc.*

Symmetry and conservation properties

Energy invariance

$$(\mathcal{C}(u,v),w) = -(v,\mathcal{C}(u,w))$$

$$\Rightarrow \quad (\mathcal{C}(u,u),u) = 0$$

Enstrophy invariance (2D)

$$(\mathcal{C}(u,v),\Delta v) = (u,\mathcal{C}(\Delta v,v))$$

$$\Rightarrow \quad ((\mathcal{S}+\mathcal{S}^*)\omega,\omega) = 0$$

Symmetry-preserving regularization

$$\tilde{\mathcal{C}}_n(\boldsymbol{u},\boldsymbol{v}) = \mathcal{C}(\boldsymbol{u},\boldsymbol{v}) + \mathcal{O}(\alpha^n)$$

Energy invariance

$$(\tilde{\mathcal{C}}_n(\boldsymbol{u},\boldsymbol{v}),\boldsymbol{w}) = -(\boldsymbol{v},\tilde{\mathcal{C}}_n(\boldsymbol{u},\boldsymbol{w}))$$

Enstrophy invariance (2D)

$$(\tilde{\mathcal{C}}_n(\boldsymbol{u},\boldsymbol{v}),\Delta\boldsymbol{v}) = (\boldsymbol{u},\tilde{\mathcal{C}}_n(\Delta\boldsymbol{v},\boldsymbol{v}))$$

Symmetry-preserving regularizations

$$\tilde{\mathcal{C}}_2(u,v) = \overline{\mathcal{C}(\overline{u},\overline{v})}$$

$$\tilde{\mathcal{C}}_4(u,v) = \mathcal{C}(\overline{u},\overline{v}) + \overline{\mathcal{C}(\overline{u},v')} + \overline{\mathcal{C}(u',\overline{v})}$$

$$\tilde{\mathcal{C}}_6(u,v) = \mathcal{C}(\overline{u},\overline{v}) + \mathcal{C}(\overline{u},v') + \mathcal{C}(u',\overline{v}) + \overline{\mathcal{C}(u',v')}$$

Energy, enstrophy (2D) and helicity are conserved

$\Rightarrow$ *Unconditional stable in energy-norm; 2D: enstrophy-norm*

Scales of motion

Eigenvalue problem diffusive operator:

$$\Pi \mathcal{D} \boldsymbol{w} = \lambda \boldsymbol{w}$$

Π *projection on* $\nabla \cdot \boldsymbol{w} = 0$

gives $0 < \lambda_0 \leq \lambda_1 \leq \lambda_2 \leq \cdots$

orthogonal basis $\{\boldsymbol{w}_j\}$

Expansion

$$\boldsymbol{u}_k = \sum_{k^2 = \lambda_j} \hat{\boldsymbol{u}}_j \boldsymbol{w}_j$$

$$\boldsymbol{u}_{k'k''} = \sum_{k' \leq k \leq k''} \boldsymbol{u}_k$$

Triadic interactions

$$\left(\frac{\mathrm{d}}{\mathrm{d}t} + \frac{|\boldsymbol{k}|^2}{\mathrm{Re}}\right)\hat{\boldsymbol{u}}_{\boldsymbol{k}} + \tilde{\boldsymbol{\mathcal{C}}}_{\boldsymbol{k}}(\hat{\boldsymbol{u}}, \hat{\boldsymbol{u}}) = \boldsymbol{0}$$

regularization

$$\tilde{\boldsymbol{\mathcal{C}}}_{\boldsymbol{k}}(\hat{\boldsymbol{u}}, \hat{\boldsymbol{v}}) = \mathrm{i}\Pi(\boldsymbol{k}) \sum_{\boldsymbol{p}+\boldsymbol{q}=\boldsymbol{k}} \hat{\boldsymbol{u}}_{\boldsymbol{p}} \boldsymbol{q} \hat{\boldsymbol{v}}_{\boldsymbol{q}} \underbrace{f(G_{\boldsymbol{k}}, G_{\boldsymbol{p}}, G_{\boldsymbol{q}})}_{\in [0,1]}$$

Large-eddies, local interactions: $\quad f = 1 - \mathcal{O}(\alpha^n)$

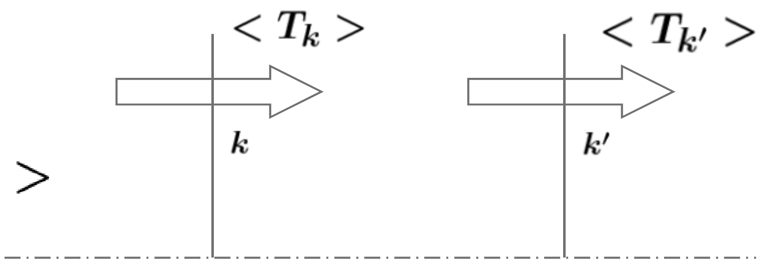
Character (forward/reverse) of transfer is maintained

Energy flux (symmetry preserving regularization)

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}|\boldsymbol{u}_{\boldsymbol{k}\boldsymbol{k}'}|^2 + \nu|\nabla \boldsymbol{u}_{\boldsymbol{k}\boldsymbol{k}'}|^2 \quad = \quad \boldsymbol{T}_{\boldsymbol{k}} - \boldsymbol{T}_{\boldsymbol{k}'}$$

Time-average:

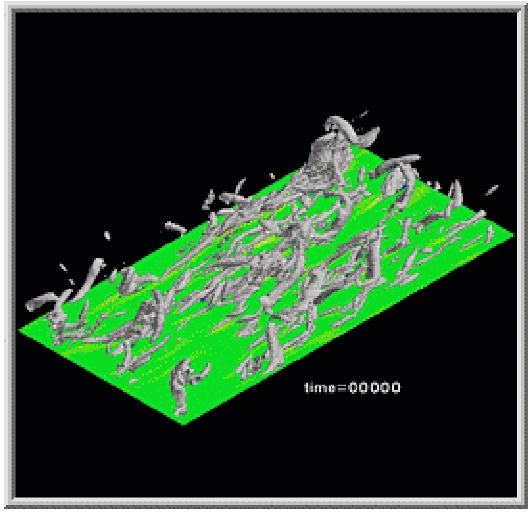
$$\nu < |\nabla \boldsymbol{u}_{\boldsymbol{k}\boldsymbol{k}'}|^2 > \; = \; < \boldsymbol{T}_{\boldsymbol{k}} > - < \boldsymbol{T}_{\boldsymbol{k}'} >$$



Same form as for Navier-Stokes eqns.

Copy proof Foias et al: ***their exists an inertial range***

$< \boldsymbol{T}_{\boldsymbol{\kappa}} >$ *is approximately constant* $\approx$ *dissipation rate*



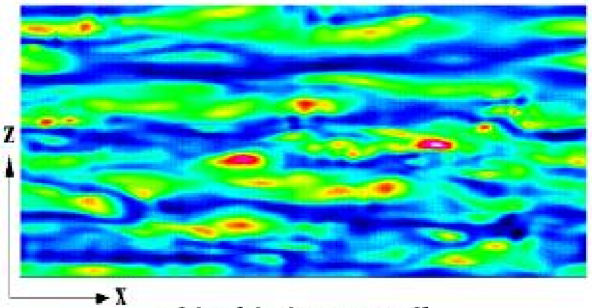
Turbulent channel flow

Turbulent flow between two parallel walls

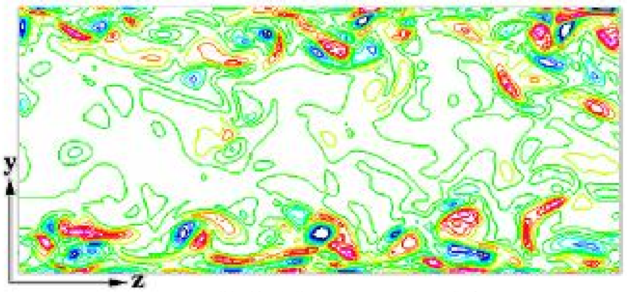
Prototype for near-wall turbulence

Break-through: DNS by KMM, 1987

Re = 5,600 4,000,000 gridpoints



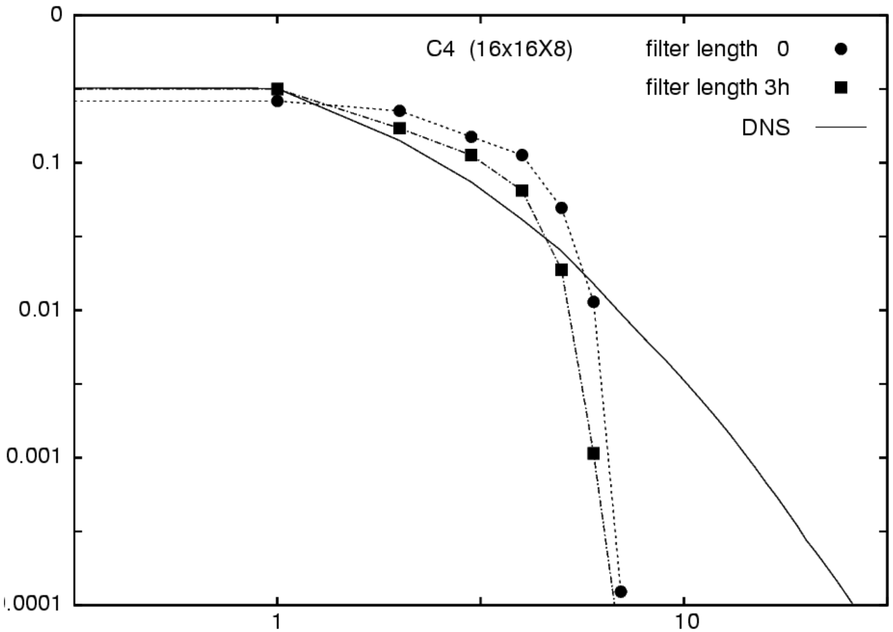
skin-friction at wall



vorticity between walls

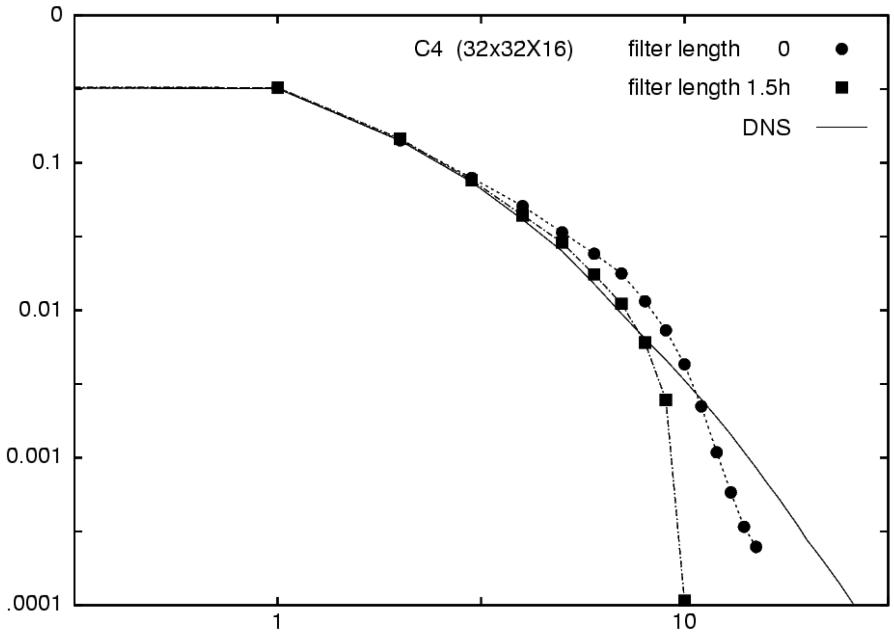
Turbulent channel flow

16x16x8 grid points



energy spectrum

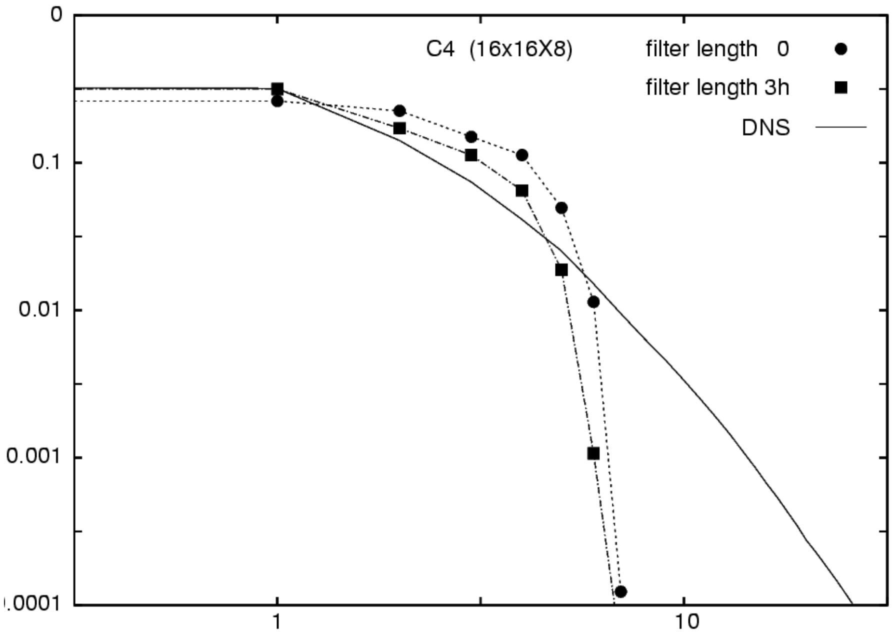
32x32x16 grid points



energy spectrum

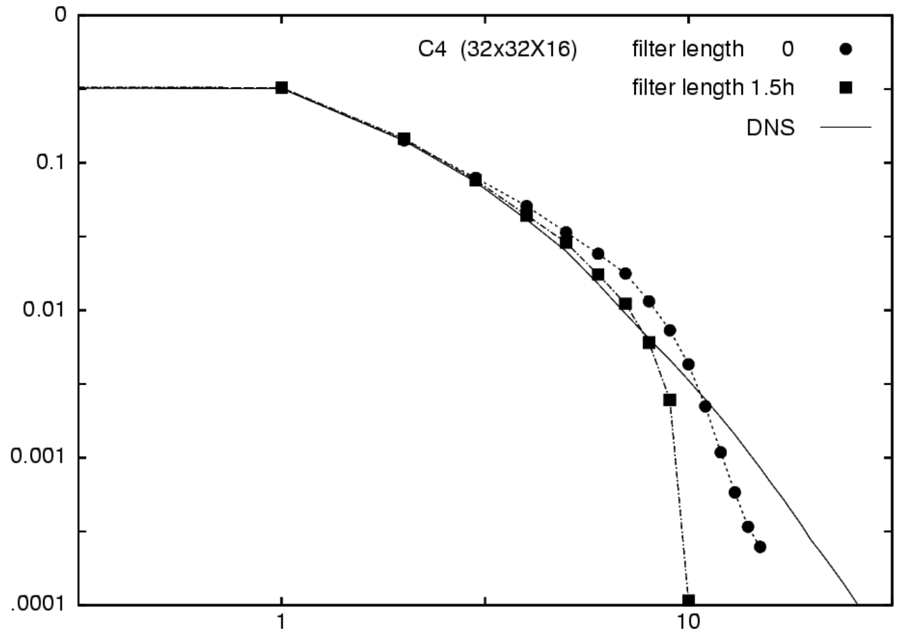
Turbulent channel flow

16x16x8 grid points



energy spectrum

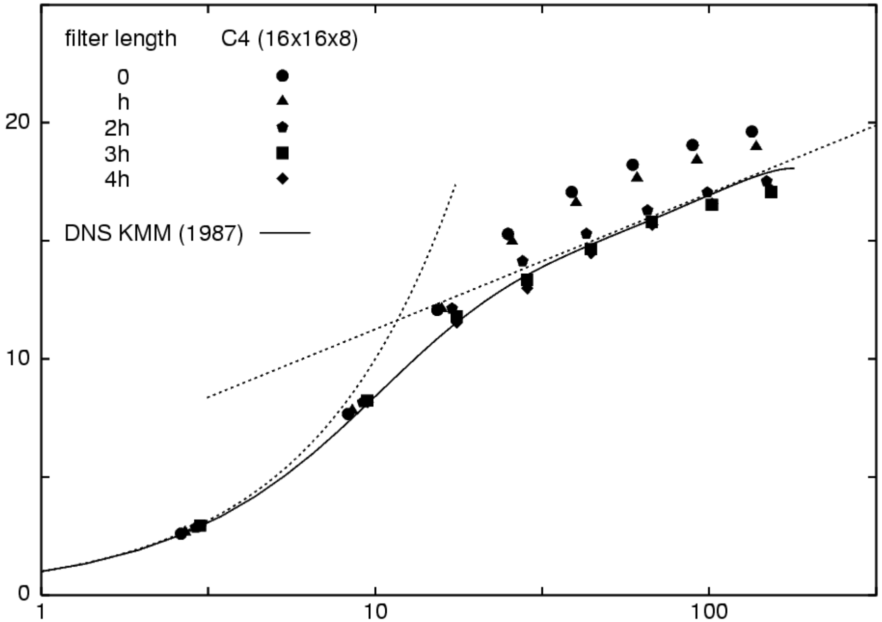
32x32x16 grid points



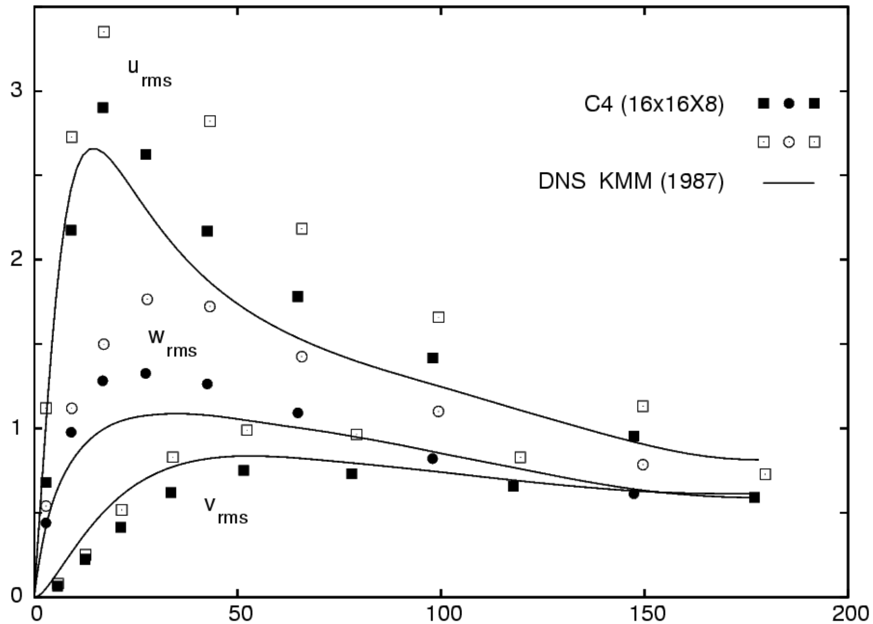
energy spectrum

Turbulent channel flow

$Re_\tau = 180$ *16x16x8 gridpoints*



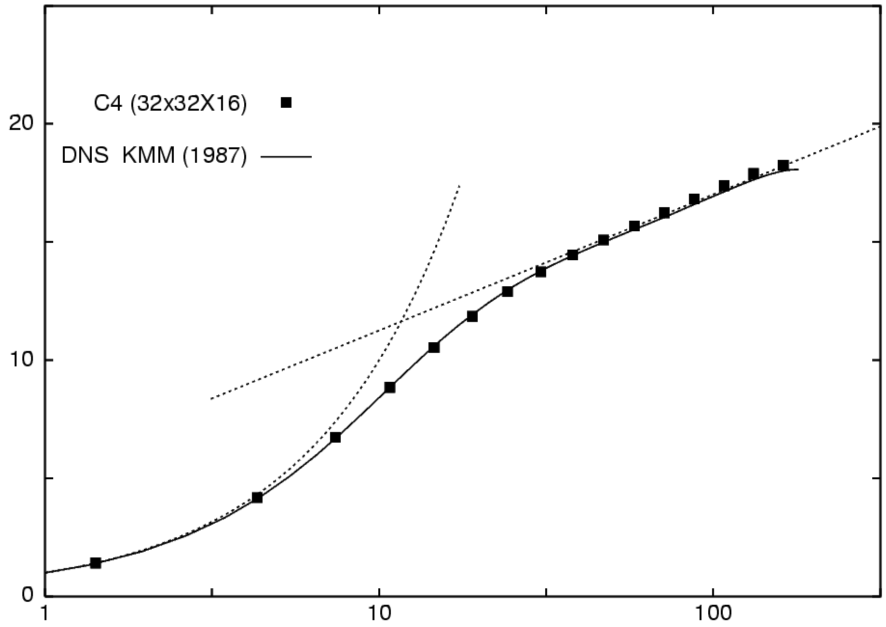
mean flow



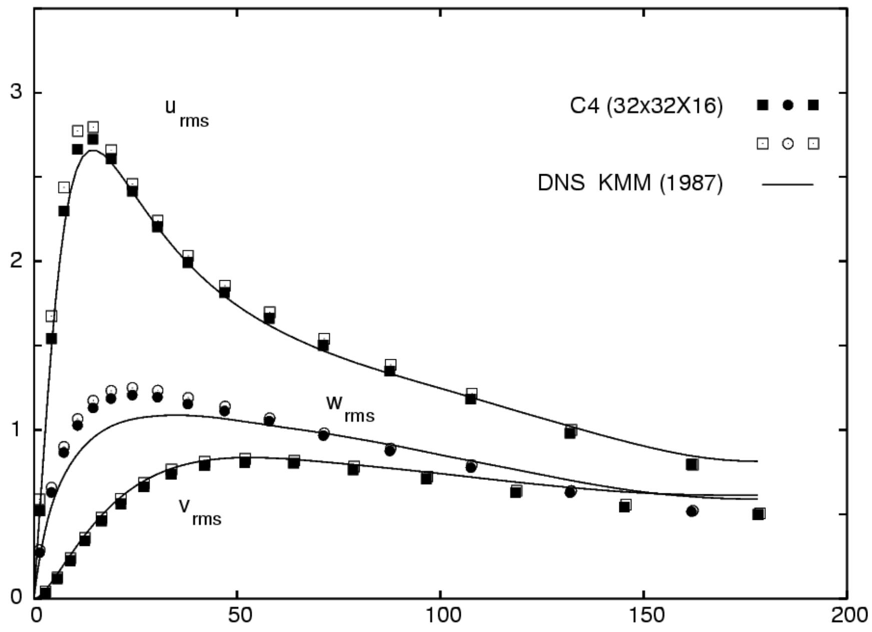
turbulence intensities

Turbulent channel flow

$Re_\tau = 180$ 32x32x16 gridpoints

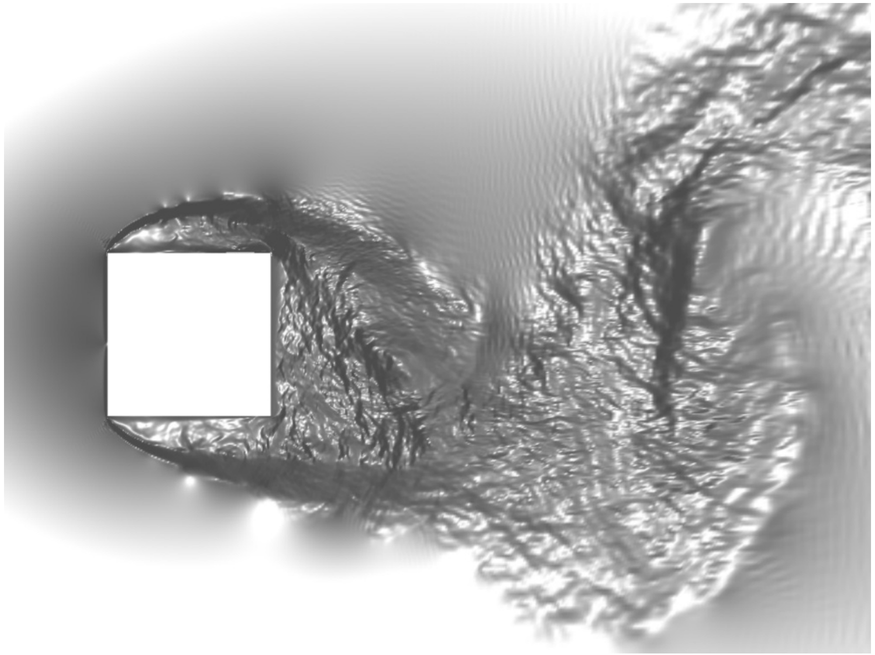


mean flow



turbulence intensities

DNS flow past square cylinder (Re=22,000)



Cartesian staggered mesh. Nodes are stretched around the cylinder.

4-th order symmetry-preserving discretization

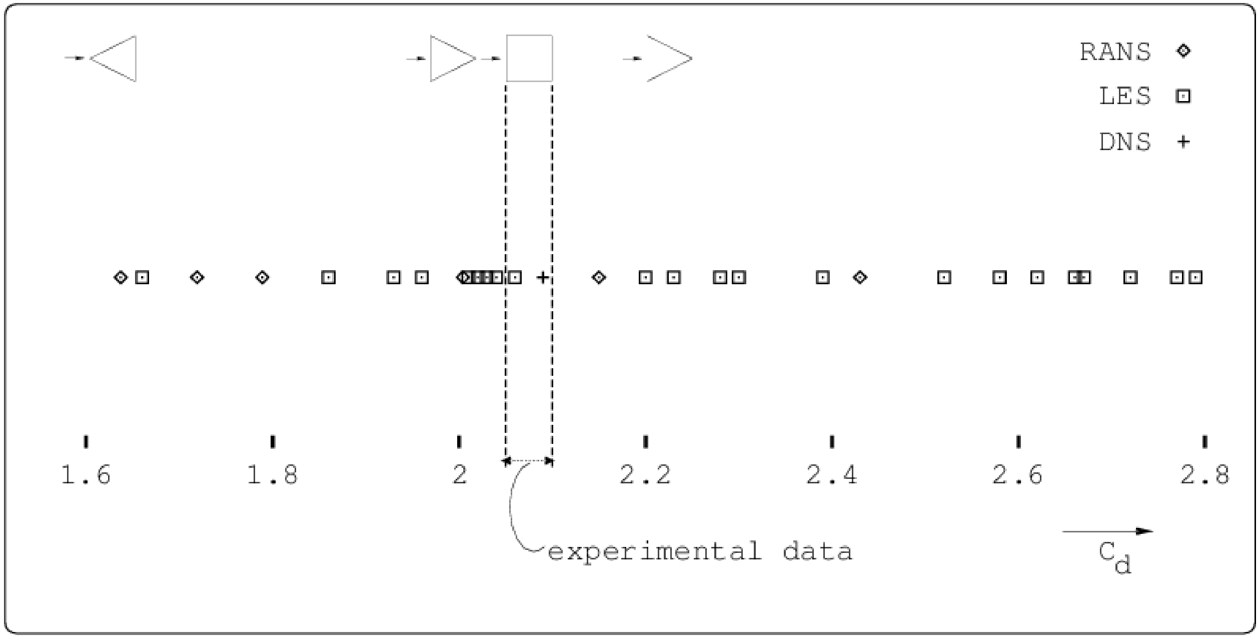
Time-integration period: **40 cycles**

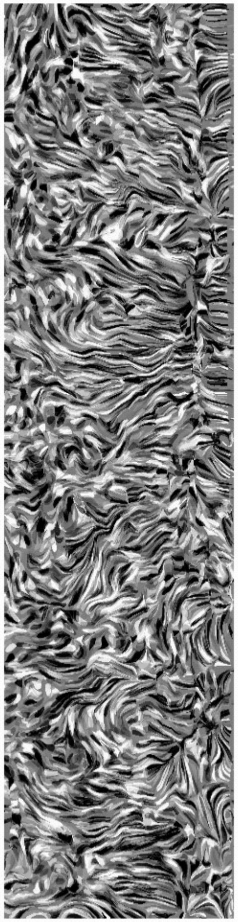
Mesh size: 128 x 1080 x 720 ~ **100M**

Computing time: ~**1 month** using **512CPUs** on the **MareNostrum** supercomputer.

DNS flow past square cylinder (Re=22,000):

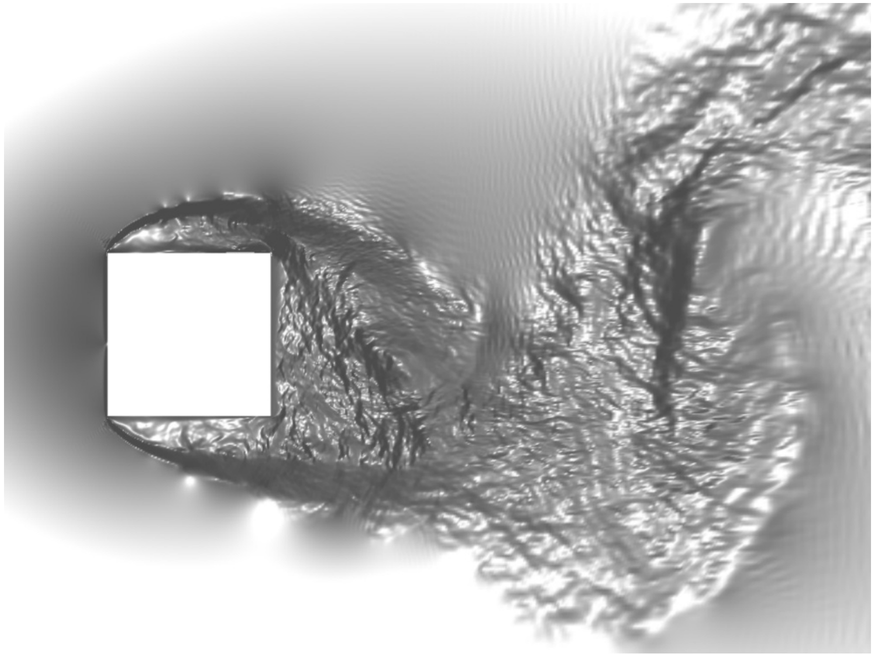
A challenge to large-eddy simulation





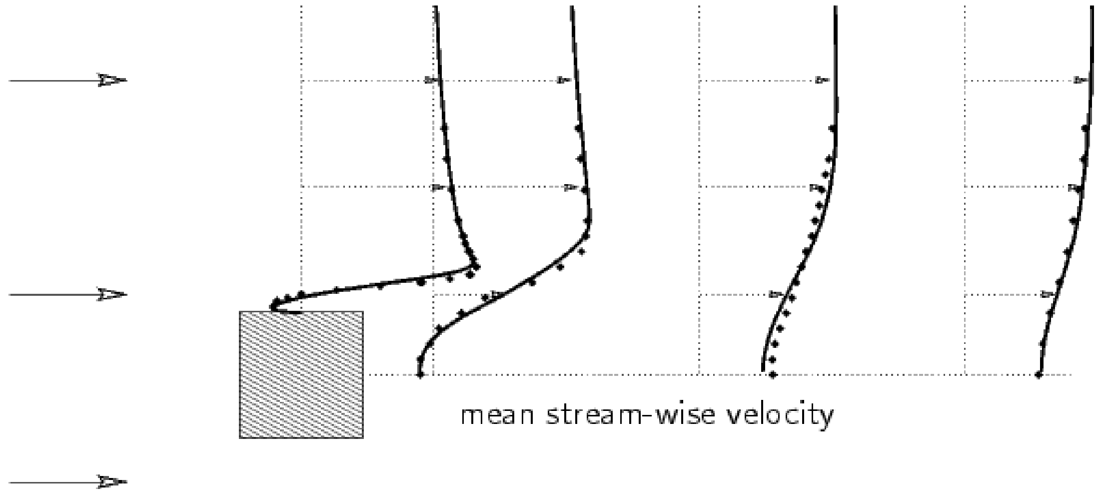
Skin-friction at cylinder

DNS flow past square cylinder, Re=22,000



	DNS	Experiments
Mean Strouhal	0.133	~0.133
Mean drag (Cd)	2.1	2.05 – 2.1
Mean lift (Cl)	0.005	~0
RMS of Cd	0.21	0.16 – 0.23
RMS of Cl	1.22	0.68 – 1.32

Mean flow profiles



- experiments by Lyn & Rodi (TU Karlsruhe)
- fourth-order symmetry-preserving method
- span = 4 diameters

Further reading

- Joop Helder and Roel Verstappen.
  On restraining convective subgrid-scale production in Burgers' equation.
  Int. J. Numer. Meth. Fluids, 56:1289-1295, 2008.

- Roel Verstappen.
  On restraining the production of small scales of motion in a turbulent channel flow.
  Comput. Fluids, 37(7):887-897, 2008.

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